A CAD SOLUTION TO THE GENERALIZED PROBLEM OF NOISE FIGURE CALCULATION

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ABSTRACT

The paper describes a new algorithm for computing the noise figure of a linear two-port network of arbitrary topology. The discussion is oriented towards CAD applications, making the approach easy to be implemented into any existing general-purpose analysis and/or design program. A few sample results are compared with measured data drawn from the technical literature.

INTRODUCTION

Present-day microwave amplifier topologies can differ substantially from the simple cascaded structure, so that the classic formula giving the combined noise figure of cascaded two-ports is no longer adequate for modern CAD purposes. Expressions for the noise figure of some popular configurations have recently appeared in the technical literature {1 - 3}. However, despite of their ingenuity, such approaches are not completely satisfactory from the standpoint of applications software, because they are based on specialized computational schemes that are only valid for preselected topologies.

The generalized problem of noise figure calculation, which is tackled in the present paper, may be stated as follows. Given a general-purpose microwave CAD program based on a component-wise circuit description, we want to build into this program an algorithm allowing the noise figure to be automatically computed, with no topology restrictions, when the circuit to be analyzed (and/or optimized) is a linear noisy two-port.

METHOD OF ANALYSIS

To develop such an algorithm, we first represent the given network (see fig. 1) as the interconnection of a lossy passive $(2m+2)$-port (named the "passive" network) with m noisy two-ports (named the "devices"). It is assumed that each device is fully characterized by means of its admittance matrix and its four spot noise parameters {4} at every frequency of interest. On the other hand, the passive network may result from an arbitrary interconnection of the usual passive microwave components; it may be nonreciprocal but its only contribution to the overall noise figure must be due to the thermal noise generators associated with ohmic losses.

To compute the noise figure, we will assume that the external ports of the network are terminated by equal resistances R_o, for this is almost always the case in microwave applications (since the passive network is completely arbitrary, the more general case of complex terminations can be treated by simply including lossless matching circuits into it). The load resistance R_o is taken as noise-free. Under such conditions, the spot noise figure of the two-port at frequency f is given by

$$F(f) = 1 + \frac{P_{INT}}{P_G} \qquad (1)$$

where

P_{INT} = active noise power delivered to the load by the noise generators acting within the network, with the input termination taken as noise-free;

P_G = active noise power delivered to the load by the equivalent noise generator of the input termination, with the entire network taken as noise-free.

The active powers appearing in (1) are those available in a narrow frequency band Δf centered around f.

As a first step we represent the noisy networks appearing in fig. 1 by their Norton equivalent circuits, that is, by their noiseless counterparts (obtained by suppressing all noise generators) with a noise current source connected across each port. This leads to the circuit configuration of fig. 2 where two sets of noise generators are shown.

The N-generators are the equivalent noise sources of the passive network and are thermal in nature; they are not statistically independent and their correlation matrix {5} has the expression

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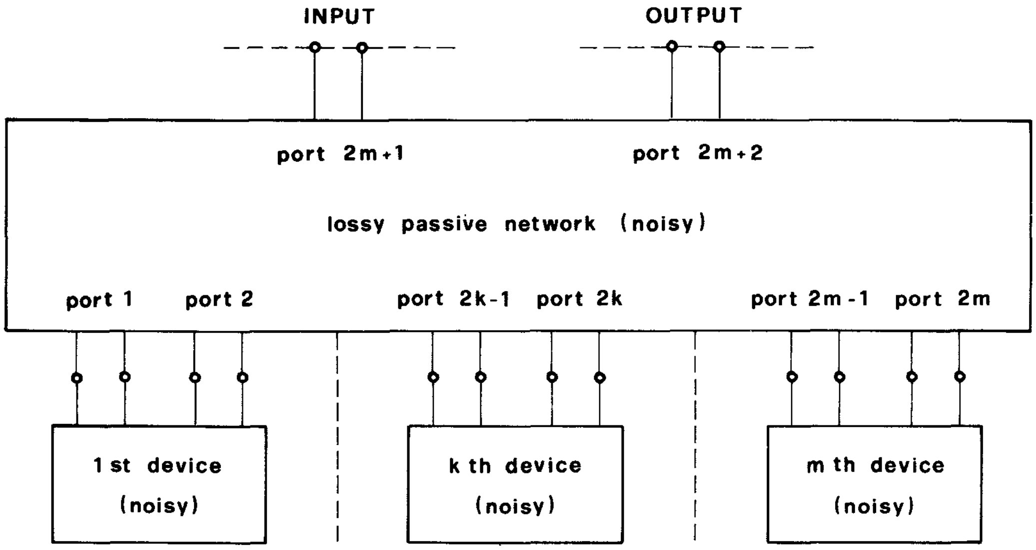

Fig. 1 - General representation of a noisy two-port.

$$\underline{\mathscr{C}}_N \equiv \left[< N_p \; N_q^* > \right] = 2\, K_B\, T_o\, \Delta f \; (\underline{Y} + \underline{Y}^*) \qquad (2)$$

$(p,\ q = 1,\ 2\ \ldots.\ 2m+2)$

where

K_B = Boltzmann's constant
T_o = absolute temperature
$\underline{Y}$ = admittance matrix of the passive network.

(< > denotes statistical average and * indicates the conjugate transposed of a complex matrix).

Note that the complex phasors N_p, N_q appearing in (2) have the meaning of pseudo-sinusoidal spectral components at frequency f of the corresponding noise currents.

The J-generators are the equivalent noise sources of the two-port devices and are usually not only thermal. Quite obviously only the couples of sources associated with the same device (namely J_{2k-1}, J_{2k} for the k-th) are correlated. Their cor relation matrix may be given the form

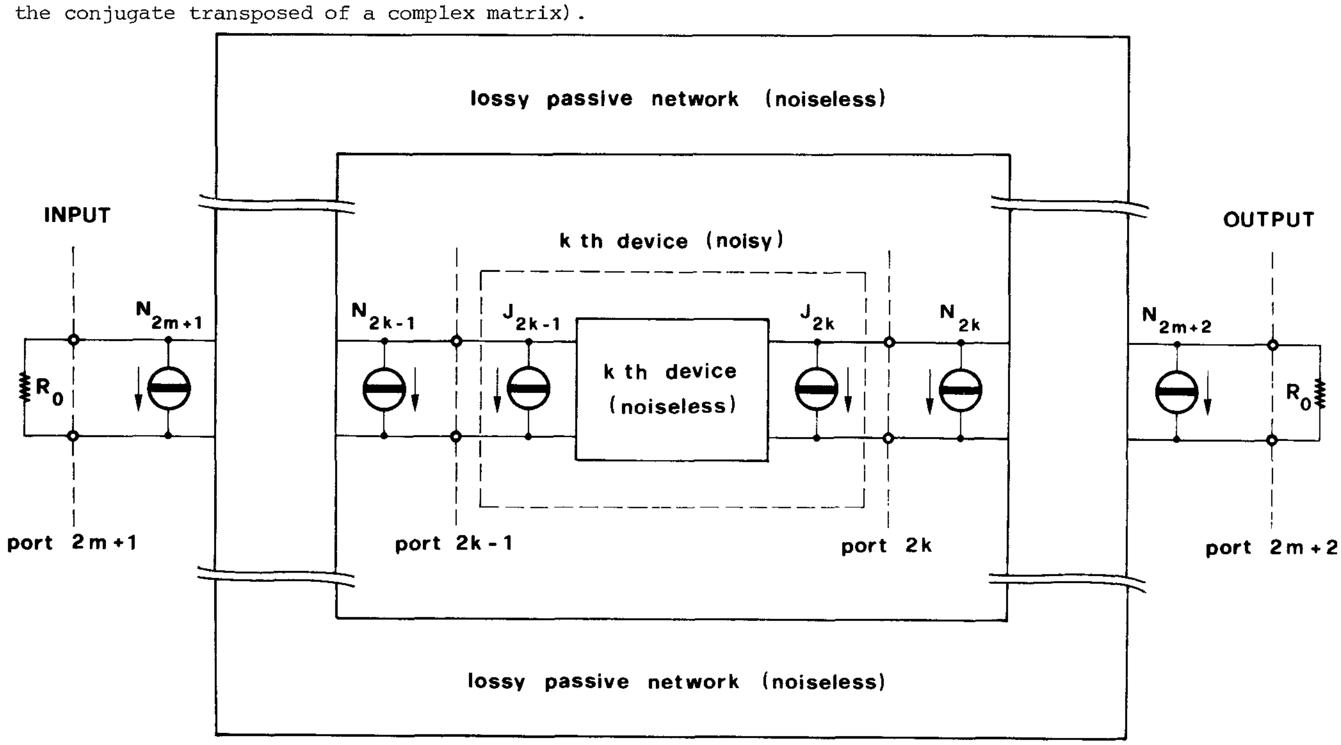

Fig. 2 - Equivalent Norton representation of a general noisy two-port.

$$\mathscr{C}_{Jk} = \left[< J_p \; J_q^* > \right] = 4\, K_B T_o \; \Delta f \; \underline{C}_{Jk} \qquad (3)$$

$$(p,\; q = 2k-1,\; 2k)$$

where $\underline{C}_{Jk}$ can be expressed as a function of the noise and admittance parameters of the k-th device making use of classic noise relationships {4}.

At this stage the evaluation of the noise figure by means of (1) is reduced to a matter of circuit algebra. The circuit of fig. 2 is analyzed in three different situations, that is,

1) with the J-sources open-circuited and a noiseless input termination;
2) with the N-sources open-circuited and a noiseless input termination;
3) with both the N- and the J-sources open-circuited and a noisy input termination.

In each case the noise voltage across the load is found and the statistical average of its squared magnitude is computed to obtain the active power delivered to the load, namely P_N, P_J and P_G, respectively. Then $P_{INT} = P_N + P_J$ because the N- and J-sources are not correlated.

For the sake of brevity the detailed calculations are not reported here; instead, the actual steps carried out by the numerical algorithm are listed below.

1) The passive network is analyzed to find its admittance matrix $\underline{Y}$. The latter is then partitioned in the form

$$\underline{Y} = \left[\begin{array}{c|c} \underline{Y}_{dd} & \underline{Y}_{de} \\ \hline \underline{Y}_{ed} & \underline{Y}_{ee} \end{array}\right] \qquad (4)$$

where the subscript d refers to the 2m device ports, and the subscript e to the 2 external ports.

2) For the k-th device $(k = 1,\; 2 \;...\; m)$, the noise correlation matrix $\underline{C}_{Jk}$ and the admittance matrix $\underline{y}_k$ (if not given) are computed, and their diagonal sums $\underline{C}_J$, $\underline{y}$ (both 2m x 2m) are built.

3) The 2 x 2m matrix

$$\underline{T} = \underline{Y}_{ed} \cdot (\underline{Y}_{dd} + \underline{y})^{-1} \qquad (5)$$

is computed.

4) The 2 x 2 admittance matrices

$$\underline{Y}_L = \underline{Y}_{ee} - \underline{T} \cdot \underline{Y}_{de}$$
$$\underline{Y}'_L = \underline{Y}_L + \frac{1}{R_o} \underline{I}_2 \qquad (6)$$

are found, and $\underline{Z}'_L = (\underline{Y}'_L)^{-1}$ is evaluated ($\underline{I}_2$ = identity matrix of order 2).

5) The row matrices

$$\underline{U}_J = [0 \;\; 1] \cdot \underline{Z}'_L \cdot \underline{T}$$
$$\underline{U}_N = \left[\underline{U}_J \;\middle|\; -[0 \;\; 1] \cdot \underline{Z}'_L \right] \qquad (7)$$

(of dimensions 1 x 2m and 1 x (2m + 2), respectively) are determined.

6) The spot noise figure is found from the expression

$$F = 1 + R_o \frac{\mathrm{Re}\left[\underline{U}_N \cdot \underline{Y} \cdot \underline{U}_N^* \right] + \underline{U}_J \cdot \underline{C}_J \cdot \underline{U}_J^*}{\left| Z'_{L21} \right|^2} \; . \qquad (8)$$

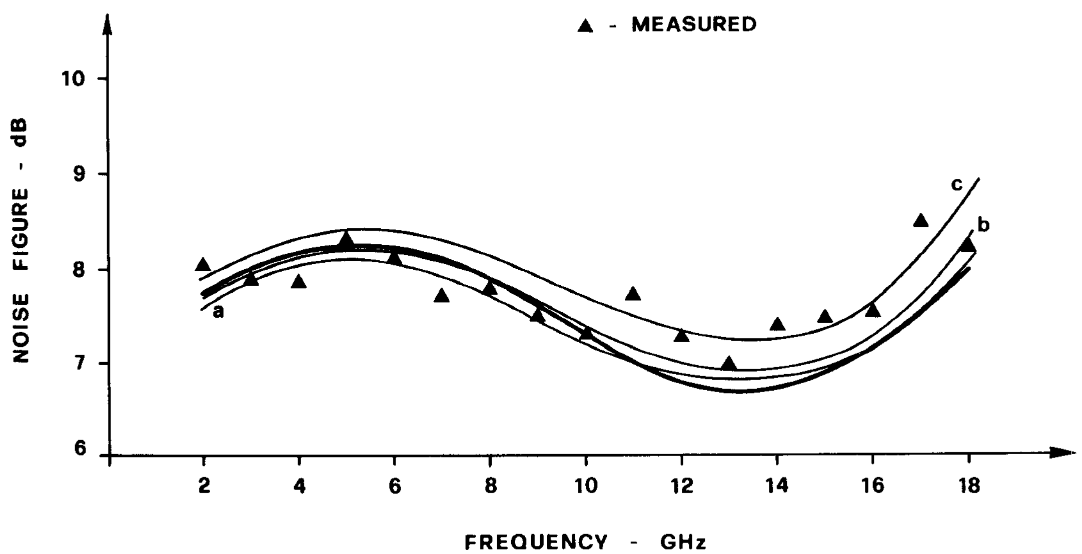

Fig. 3 - Noise figure of a distributed amplifier. The thick solid line was computed by Niclas ans Tucker {2}. The thin lines were obtained by the present method with a) $R_s = 0$; b) $R_s = 0.1\ \Omega$; c) $R_s = 0.2\ \Omega$.

Note that $\underline{Y}_L$ given by the first of (6) is the admittance matrix of the overall 2-port network, so that a conventional circuit analysis is automatically carried out as a by-product of the noise figure calculation. This allows a straightforward optimization of an absolutely general amplifier topology with respect to both noise figure and any of the conventional network functions. An interesting feature of (8) is that it provides separate explicit expressions for the two main contributions to the overall noise figure, that is, the thermal noise of the passive network and the noise injected by the active devices.

RESULTS

The above algorithm was implemented into a general-purpose microwave CAD program based on the Subnetwork-Growth Method (SGM) of network analysis {6}.
When the noise figure is one of the network functions selected by the user, a preliminary section of the program (running only once for a given design) eliminates the devices from the user-defined topology thus generating the passive network of fig. 1. The latter is then analyzed in the usual way, and a special subroutine is called to perform the steps of the algorithm. In this way the noise figure calculation becomes absolutely transparent to the user no matter what the circuit topology. When the noise figure is not required the special sections of the program are bypassed, and a conventional analysis of the given two-port is carried out.
A peculiar advantage of the SGM is that the same subroutines can be used in both cases to carry out the interconnection of the circuit components. Thus only minor changes have to be made in an existing program in order to implement the noise figure calculation.

As a practical example the 3-stage distributed amplifier described by Niclas and Tucker {2} was analyzed by the above method.
Numerical results are shown in fig. 3 together with the experimental results reported in {2}. Several curves are given in the figure depending on the skin resistance of the microstrip lines at 10 GHz, namely R_S, as a parameter. Microstrip losses were evaluated by the formulae given in {7}. For $R_S = 0$ the theoretical curve presented in {2} is approximately reobtained (curve a). Minor discrepancies (of the order of 0.2 dB) are probably due to the use of slightly different microstrip models.
The curves obtained for $R_S = 0.1$ and 0.2 Ω are also shown in the figure. These results support the statement {2} that microstrip losses give a practically negligible contribution to the amplifier noise figure: in fact, a typical value for R_S is around 0.05 Ω for the technology used to build this particular amplifier.

ACKNOWLEDGMENT

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